PRACTICE FINAL EXAM

Math 340
12/15/2017

Name: ____________________________

ID: ____________________________

"My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this"

Signature: ____________________________

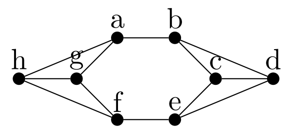
Read all of the following information before starting the exam:

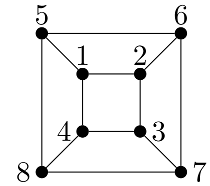
- Check your exam to make sure all pages are present.
- Leave answers unsimplified—you may include factorials, $P(n, k)$, $\binom{n}{k}$, etc. in your answers. Do not leave unevaluated $\sum_{i \leq k}$ in your final answers.
- You may use writing implements and a single 3"x5" notecard.
- You may not use a calculator.
- You may use any result proved in class or in the textbook in your arguments.
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- Good luck!

1	15	
2	10	
3	10	
4	15	
5	12	
6	12	
7	14	
Total	100	

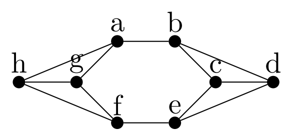
1. (*15 points*) For each of the following pairs of graphs, explain how you can be sure the pair is not isomorphic. (All the graphs have eight vertices and every vertex has degree 3.)

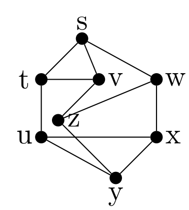
(**a**)



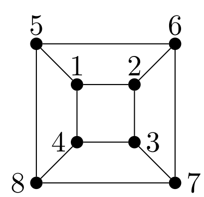


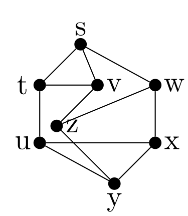
(**b**)





(**c**)





2. (*10 points*) Suppose G is a connected graph such that every vertex has degree at least 5 and which contains no triangles.

(**a**) If **e** is the number of edges and **v** is the number of vertices, give an upper bound for **v** in terms of **e**. (That is, find something to go in the blank in $\mathbf{v} \leq$ ___.)

(**b**) Suppose that we are able to draw G in the plane and that **r** is the number of regions in this drawing. Give an upper bound on **r** in terms of **e**.

(**c**) Use Euler's formula, $\mathbf{r} = \mathbf{e} - \mathbf{v} + 2$, to show that G is not planar.

3. (*15 points*) (**a**) How many ways are there to rearrange the letters in the word PLATYPUS so that the vowels A and U are not adjacent?

(**b**) How many ways are there to rearrange the letters in the word PLATYPUS so that the vowels A and U are not adjacent and the letter Y comes after the letter A?

4. (*14 points*) Suppose we make a 9 digit ternery sequence (i.e. using the digits $\{0, 1, 2\}$). How many sequences are there where the subsequence 012 (those three values in a row in that order) never appears? (Hint: use inclusion-exclusion, with the set A_i being the number of sequences where the subsequence 012 does appear, starting in the i-th position; for instance, A_1 is the number of 9 digit sequences which start with 012.)

5. (*12 points*) There are 6 place settings around a circular table. At each setting is a bowl of ice cream, chosen from one of 31 available flavors.

(**a**) If each place setting is distinct, how many possible arrangements of ice cream flavors are there?

(**b**) If we treat rotations as symmetric, how many possible arrangements are there? (So if we rotate all the ice creams one step clockwise, that counts as the same arrangement.)

6. (*10 points*) We are interested in ternery sequences (i.e. using the digits $\{0,1,2\}$) which never contain two consecutive 0's. (So 121, 010, and 011 are allowed sequences, but 100 is not.) Let s_n be the number of such sequences of length n.

(**a**) How many of these sequences are there of length 1?

(**b**) How many of these sequences are there of length 2?

(**c**) Verify that the numbers s_n satisfy the recurrence relation

$$s_n = 2(s_{n-1} + s_{n-2}).$$

(**d**) Solve the recurrence relation to give a formula for s_n.

7. (*10 points*) Grundy's Game is, like Nim, a two-player game played with several heaps, where each heap has some number of pennies. In Grundy's Game, the only available move is splitting an existing heap into two smaller heaps which have two different sizes. The game ends when all heaps have size 1 or 2, and therefore cannot be further split; the player who made the last split wins.
What is the Grundy number (nimber) of the game starting with a single heap with five pennies?